

# 4-7

## Congruence in Overlapping Triangles

### Common Core State Standards

**G-SRT.B.5** Use congruence . . . criteria to solve problems and prove relationships in geometric figures.

**MP 1, MP 3, MP 4**

**Objectives** To identify congruent overlapping triangles  
To prove two triangles congruent using other congruent triangles



Do all the triangles make you dizzy? Try to see each one. Then learn some tricks that may help you.



### Getting Ready!

An assignment for your graphic design class is to make a colorful design using triangles. How many triangles are in your design? Explain how you count them.



### MATHEMATICAL PRACTICES

In the Solve It, you located individual triangles among a jumble of triangles. Some triangle relationships are difficult to see because the triangles overlap.

**Essential Understanding** You can sometimes use the congruent corresponding parts of one pair of congruent triangles to prove another pair of triangles congruent. This often involves overlapping triangles.

Overlapping triangles may have a common side or angle. You can simplify your work with overlapping triangles by separating and redrawing the triangles.

### Think

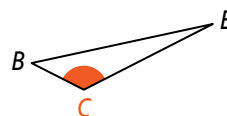
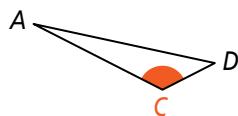
How can you see an individual triangle in order to redraw it? Use your finger to trace along the lines connecting the three vertices. Then cover up any untraced lines.



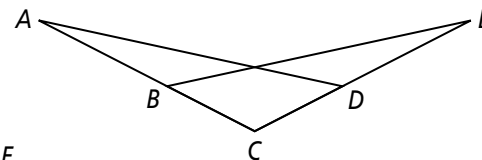
### Problem 1 Identifying Common Parts

What common angle do  $\triangle ACD$  and  $\triangle ECB$  share?

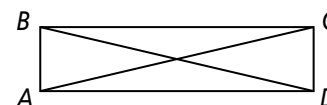
Separate and redraw  $\triangle ACD$  and  $\triangle ECB$ .



The common angle is  $\angle C$ .



- Got It?** 1. a. What is the common side in  $\triangle ABD$  and  $\triangle DCA$ ?  
b. What is the common side in  $\triangle ABD$  and  $\triangle BAC$ ?

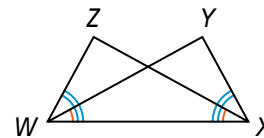




**Proof** **Problem 2** Using Common Parts

**Given:**  $\angle ZXW \cong \angle YWX$ ,  $\angle ZWX \cong \angle YXW$

**Prove:**  $\overline{ZW} \cong \overline{YX}$

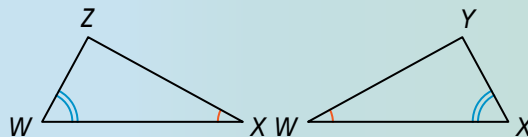


**Know**

- $\angle ZXW \cong \angle YWX$  and  $\angle ZWX \cong \angle YXW$
- The diagram shows that  $\triangle ZWX$  and  $\triangle YXW$  are overlapping triangles.

**Need**

A diagram of the triangles separated



**Plan**

Show  $\triangle ZWX \cong \triangle YXW$ . Then use corresponding parts of congruent triangles to prove  $\overline{ZW} \cong \overline{YX}$ .

$\angle ZXW \cong \angle YWX$

Given

$\overline{WX} \cong \overline{WX}$

Reflexive Prop. of  $\cong$

$\angle ZWX \cong \angle YXW$

Given

$\triangle ZWX \cong \triangle YXW$

ASA

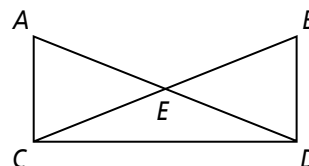
$\overline{ZW} \cong \overline{YX}$

Corresp. parts of  $\cong \triangle$  are  $\cong$ .



**Got It? 2. Given:**  $\triangle ACD \cong \triangle BDC$

**Prove:**  $\overline{CE} \cong \overline{DE}$

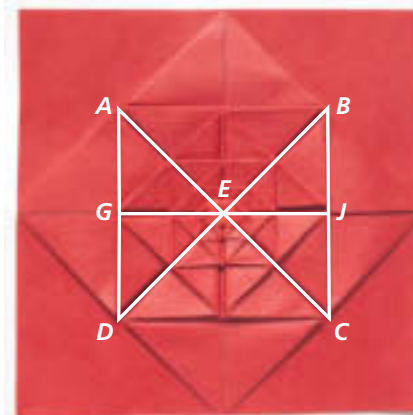


**Proof** **Problem 3** Using Two Pairs of Triangles

**Given:** In the origami design,  $E$  is the midpoint of  $\overline{AC}$  and  $\overline{DB}$ .

**Prove:**  $\triangle GED \cong \triangle JEB$

**Proof:**  $E$  is the midpoint of  $\overline{AC}$  and  $\overline{DB}$ , so  $\overline{AE} \cong \overline{CE}$  and  $\overline{DE} \cong \overline{BE}$ .  $\angle AED \cong \angle CEB$  because vertical angles are congruent. Therefore,  $\triangle AED \cong \triangle CEB$  by SAS.  $\angle D \cong \angle B$  because corresponding parts of congruent triangles are congruent.  $\angle GED \cong \angle JEB$  because vertical angles are congruent. Therefore,  $\triangle GED \cong \triangle JEB$  by ASA.



**Plan**

How do you choose another pair of triangles to help in your proof?

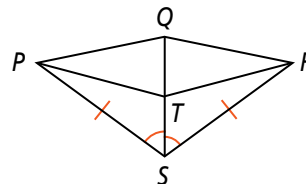
Look for triangles that share parts with  $\triangle GED$  and  $\triangle JEB$  and that you can prove congruent. In this case, first prove  $\triangle AED \cong \triangle CEB$ .





**Got It? 3. Given:**  $\overline{PS} \cong \overline{RS}$ ,  $\angle PSQ \cong \angle RSQ$

**Prove:**  $\triangle QPT \cong \triangle QRT$



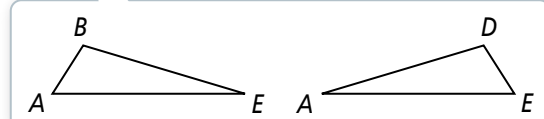
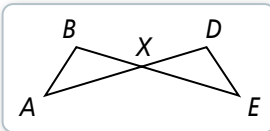
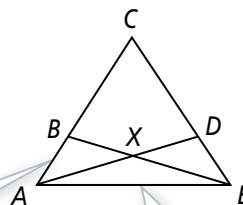
When several triangles overlap and you need to use one pair of congruent triangles to prove another pair congruent, you may find it helpful to draw a diagram of each pair of triangles.



### Problem 4 Separating Overlapping Triangles

**Given:**  $\overline{CA} \cong \overline{CE}$ ,  $\overline{BA} \cong \overline{DE}$

**Prove:**  $\overline{BX} \cong \overline{DX}$



### Plan

Which triangles are useful here?

If  $\triangle BXA \cong \triangle DXE$ , then  $\overline{BX} \cong \overline{DX}$  because they are corresponding parts. If  $\triangle BAE \cong \triangle DEA$ , you will have enough information to show  $\triangle BXA \cong \triangle DXE$ .

#### Statements

- 1)  $\overline{BA} \cong \overline{DE}$
- 2)  $\overline{CA} \cong \overline{CE}$
- 3)  $\angle CAE \cong \angle CEA$
- 4)  $\overline{AE} \cong \overline{AE}$
- 5)  $\triangle BAE \cong \triangle DEA$
- 6)  $\angle ABE \cong \angle EDA$
- 7)  $\angle BXA \cong \angle DXE$
- 8)  $\triangle BXA \cong \triangle DXE$
- 9)  $\overline{BX} \cong \overline{DX}$

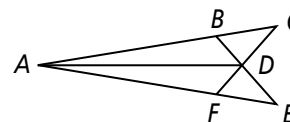
#### Reasons

- 1) Given
- 2) Given
- 3) Base  $\angle$ s of an isosceles  $\triangle$  are  $\cong$ .
- 4) Reflexive Property of  $\cong$
- 5) SAS
- 6) Corresp. parts of  $\cong \triangle$  are  $\cong$ .
- 7) Vertical angles are  $\cong$ .
- 8) AAS
- 9) Corresp. parts of  $\cong \triangle$  are  $\cong$ .



**Got It? 4. Given:**  $\angle CAD \cong \angle EAD$ ,  $\angle C \cong \angle E$

**Prove:**  $\overline{BD} \cong \overline{FD}$





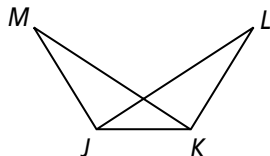


## Lesson Check

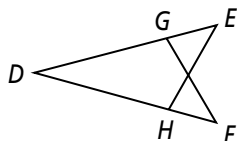
### Do you know HOW?

Identify any common angles or sides.

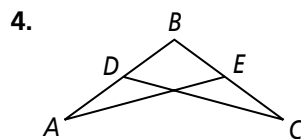
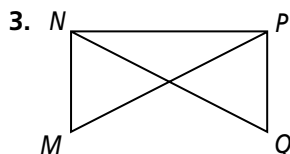
1.  $\triangle MKJ$  and  $\triangle LJK$



2.  $\triangle DEH$  and  $\triangle DFG$



Separate and redraw the overlapping triangles. Label the vertices.

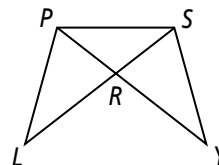


### Do you UNDERSTAND?

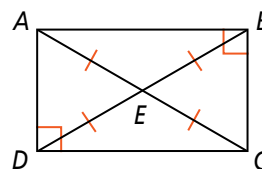


5. **Reasoning** In Exercise 1, both triangles have vertices  $J$  and  $K$ . Are  $\angle J$  and  $\angle K$  common angles for  $\triangle MKJ$  and  $\triangle LJK$ ? Explain.

6. **Error Analysis** In the diagram,  $\triangle PSY \cong \triangle SPL$ . Based on that fact, your friend claims that  $\triangle PRL \cong \triangle SRY$ . Explain why your friend is incorrect.



7. In the figure below, which pair of triangles could you prove congruent first in order to prove that  $\triangle ACD \cong \triangle CAB$ ? Explain.



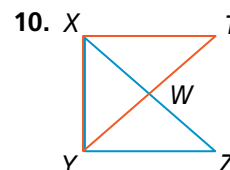
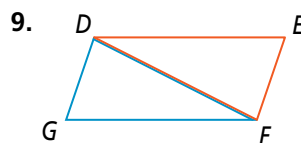
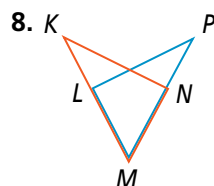
## Practice and Problem-Solving Exercises



### A Practice

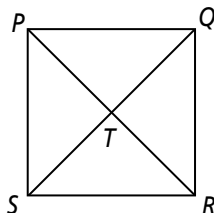
In each diagram, the red and blue triangles are congruent. Identify their common side or angle.

See Problem 1.

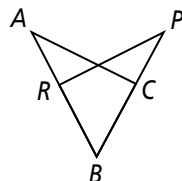


Separate and redraw the indicated triangles. Identify any common angles or sides.

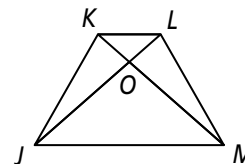
11.  $\triangle PQS$  and  $\triangle QPR$



12.  $\triangle ACB$  and  $\triangle PRB$



13.  $\triangle JKL$  and  $\triangle MLK$

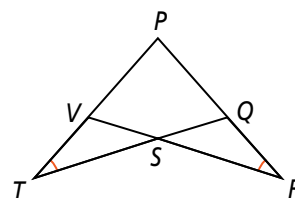




14. **Developing Proof** Complete the flow proof.

**Given:**  $\angle T \cong \angle R$ ,  $\overline{PQ} \cong \overline{PV}$

**Prove:**  $\angle PQT \cong \angle PVR$



$\angle T \cong \angle R$

a. ?

$\angle TPQ \cong \angle RPV$

b. ?

$\overline{PQ} \cong \overline{PV}$

c. ?

$\triangle TPQ \cong \triangle RPV$

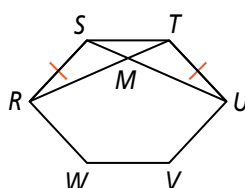
d. ?

$\angle PQT \cong \angle PVR$

e. ?

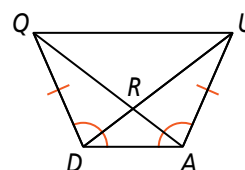
15. **Given:**  $\overline{RS} \cong \overline{UT}$ ,  $\overline{RT} \cong \overline{US}$

**Proof** **Prove:**  $\triangle RST \cong \triangle UTS$



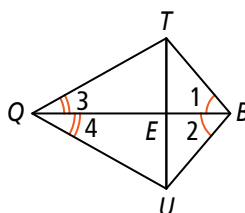
16. **Given:**  $\overline{QD} \cong \overline{UA}$ ,  $\angle QDA \cong \angle UAD$

**Proof** **Prove:**  $\triangle QDA \cong \triangle UAD$



17. **Given:**  $\angle 1 \cong \angle 2$ ,  $\angle 3 \cong \angle 4$

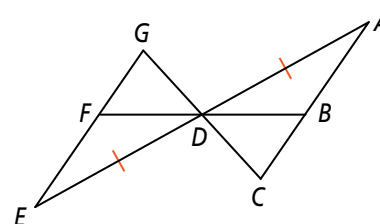
**Proof** **Prove:**  $\triangle QET \cong \triangle QEU$



18. **Given:**  $\overline{AD} \cong \overline{ED}$ , **See Problems 3 and 4.**

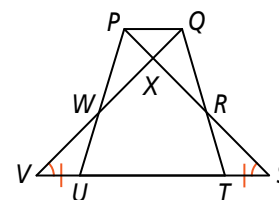
**Proof** D is the midpoint of  $\overline{BF}$

**Prove:**  $\triangle ADC \cong \triangle EDG$



19. **Think About a Plan** In the diagram at the right,  $\angle V \cong \angle S$ ,  $\overline{VU} \cong \overline{ST}$ , and  $\overline{PS} \cong \overline{QV}$ . Which two triangles are congruent by SAS? Explain.

- How can you use a new diagram to help you identify the triangles?
- What do you need to prove triangles congruent by SAS?

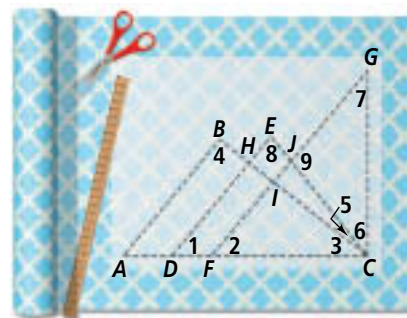




- STEM 20. Clothing Design** The figure at the right is part of a clothing design pattern, and it has the following relationships.

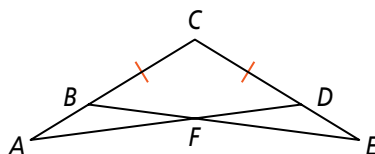
- $\overline{GC} \perp \overline{AC}$
- $\overline{AB} \perp \overline{BC}$
- $\overline{AB} \parallel \overline{DE} \parallel \overline{FG}$
- $m\angle A = 50$
- $\triangle DEC$  is isosceles with base  $\overline{DC}$ .

- Find the measures of all the numbered angles in the figure.
- Suppose  $\overline{AB} \cong \overline{FC}$ . Name two congruent triangles and explain how you can prove them congruent.



- 21. Given:**  $\overline{AC} \cong \overline{EC}$ ,  $\overline{CB} \cong \overline{CD}$

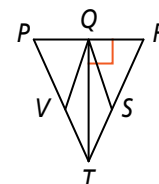
**Proof** **Prove:**  $\angle A \cong \angle E$



- 22. Given:**  $\overline{QT} \perp \overline{PR}$ ,  $\overline{QT}$  bisects  $\overline{PR}$ ,

**Proof**  $\overline{QT}$  bisects  $\angle VQS$

**Prove:**  $\overline{VQ} \cong \overline{SQ}$



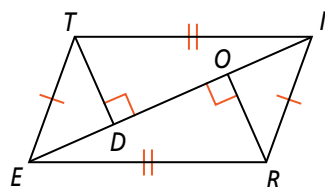
**Open-Ended** Draw the diagram described.

- 23.** Draw a vertical segment on your paper. On the right side of the segment draw two triangles that share the vertical segment as a common side.

- 24.** Draw two triangles that have a common angle.

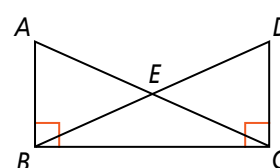
- 25. Given:**  $\overline{TE} \cong \overline{RI}$ ,  $\overline{TI} \cong \overline{RE}$ ,  
**Proof**  $\angle TDI$  and  $\angle ROE$  are right  $\angle$ s

**Prove:**  $\overline{TD} \cong \overline{RO}$



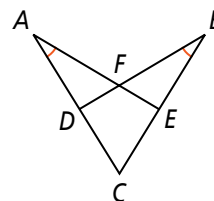
- 26. Given:**  $\overline{AB} \perp \overline{BC}$ ,  $\overline{DC} \perp \overline{BC}$ ,  
**Proof**  $\overline{AC} \cong \overline{DB}$

**Prove:**  $\overline{AE} \cong \overline{DE}$



- Challenge 27.** Identify a pair of overlapping congruent triangles in the diagram. Then use the given information to write a proof to show that the triangles are congruent.

**Given:**  $\overline{AC} \cong \overline{BC}$ ,  $\angle A \cong \angle B$





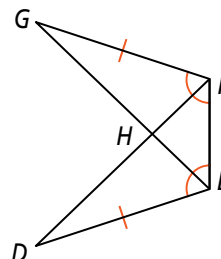
- 28. Reasoning** Draw a quadrilateral  $ABCD$  with  $\overline{AB} \parallel \overline{DC}$ ,  $\overline{AD} \parallel \overline{BC}$ , and diagonals  $\overline{AC}$  and  $\overline{DB}$  intersecting at  $E$ . Label your diagram to indicate the parallel sides.
- List all the pairs of congruent segments in your diagram.
  - Writing** Explain how you know that the segments you listed are congruent.

## Standardized Test Prep

SAT/ACT

29. According to the diagram at the right, which statement is true?

- (A)  $\triangle DEH \cong \triangle GFH$  by AAS  
 (B)  $\triangle DEH \cong \triangle GFH$  by SAS  
 (C)  $\triangle DEF \cong \triangle GFE$  by AAS  
 (D)  $\triangle DEF \cong \triangle GFE$  by SAS



30.  $\triangle ABC$  is isosceles with base  $\overline{AC}$ . If  $m\angle C = 37$ , what is  $m\angle B$ ?

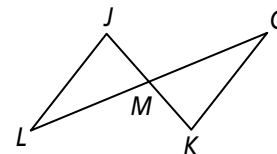
- (F) 37                      (G) 74                      (H) 106                      (I) 143

31. Which word correctly completes the statement “All   ? angles are congruent”?

- (A) adjacent              (B) supplementary              (C) right              (D) corresponding

32. In the figure,  $\overline{LJ} \parallel \overline{GK}$  and  $M$  is the midpoint of  $\overline{LG}$ .

- Copy the diagram. Then mark your diagram with the given information.
- Prove  $\triangle LJM \cong \triangle GKM$ .
- Can you prove that  $\triangle LJM \cong \triangle GKM$  another way? Explain.



Extended Response

## Mixed Review

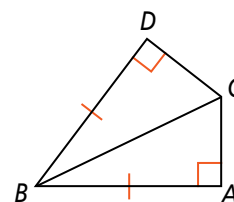
- 33. Developing Proof** Complete the paragraph proof.

**Given:**  $\overline{AB} \cong \overline{DB}$ ,  $\angle A$  and  $\angle D$  are right angles

**Prove:**  $\triangle ABC \cong \triangle DBC$

**Proof:** You are given that  $\overline{AB} \cong \overline{DB}$  and  $\angle A$  and  $\angle D$  are right angles.  $\triangle ABC$  and  $\triangle DBC$  are **a. ?** triangles by the definition of **b. ?** triangle.  $\overline{BC} \cong \overline{BC}$  by the **c. ?** Property of Congruence.  $\triangle ABC \cong \triangle DBC$  by the **d. ?** Theorem.

See Lesson 4-6.



34. **Constructions** Draw a line  $p$  and a point  $M$  not on  $p$ . Then construct line  $n$  through  $M$  so that  $n \perp p$ .

See Lesson 3-6.

**Get Ready!** To prepare for Lesson 5-1, do Exercises 35–37.

Find the coordinates of the midpoint of  $\overline{AB}$ .

See Lesson 1-7.

35.  $A(-2, 3)$ ,  $B(4, 1)$

36.  $A(0, 5)$ ,  $B(3, 6)$

37.  $A(7, 10)$ ,  $B(-5, -8)$